10:35-11:10 Tomasz Klimsiak (Nicolaus Copernicus University in Toruń, Poland)

Dynkin games and related PDEs

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Suppose that $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space equipped with a right-continuous filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ satisfying the usual conditions, and let T be an $\mathbb{F}$-stopping time taking values in $[0, \infty]$. Let ξ be an integrable $\mathcal{F}_T$-measurable random variable, let

$$f : \Omega \times \mathbb{R}_+ \times \mathbb{R} \longrightarrow \mathbb{R}$$

be progressively measurable with respect to (ω, t), and let L and U be $\mathbb{F}$-adapted càdlàg processes of class (D) such that $L \leq U$ and

$$\limsup_{a\to\infty} L_{T\wedge a} \leq \xi \leq \liminf_{a\to\infty} U_{T\wedge a}, \qquad \mathbb{P}\text{-a.s.}$$

The talk concerns reflected backward stochastic differential equations (RBSDEs) of the form

$$dY_t = -f(t, Y_t)\, dt - dR_t + dM_t, \qquad L \leq Y \leq U, \qquad Y_{T\wedge a} \longrightarrow \xi \quad \text{as } a \to \infty,$$

and their connections with nonlinear Dynkin games and nonlinear partial differential equations. In the classical theory, a crucial part of the definition of a solution to (1) is the Skorokhod condition

$$\int_0^T (Y_{t-} - L_{t-})\, dR_t^+ = \int_0^T (U_{t-} - Y_{t-})\, dR_t^- = 0,$$

where $R = R^+ - R^-$ is a predictable process of finite variation. This condition entails that every solution has a semimartingale structure. Consequently, under suitable assumptions on the generator, the classical well-posedness theory for RBSDEs with two barriers is typically formulated under Mokobodzki's condition on L and U. On the other hand, for the nonlinear f-expectation $\mathbb{E}^f$, it is known that, in certain instances, the family of random variables

$$Y_\alpha := *ess\,sup_{\sigma\geq\alpha} * ess\,inf_{\tau\geq\alpha} \mathbb{E}^f_{\alpha,\tau\wedge\sigma} \left(L_\sigma \mathbf{1}_{\{\sigma<\tau\}} + U_\tau \mathbf{1}_{\{\tau\leq\sigma<T\}} + \xi \mathbf{1}_{\{\sigma=\tau=T\}} \right)$$

admits a càdlàg aggregation even when Mokobodzki's condition fails. This raises the natural question of whether this process can be characterized as the unique solution of a suitably extended reflected BSDE. We introduce such an extended notion of solution, establish existence and uniqueness results, and prove a one-to-one correspondence between solutions of reflected BSDEs and value processes of nonlinear Dynkin games. Finally, we discuss connections between this framework and nonlinear partial differential equations. The talk is based on [1] and [2].

References:

[1] Klimsiak, T.: Non-semimartingale solutions of reflected BSDEs and applications to Dynkin games. *Stochastic Process. Appl.* **134** (2021) 208-239

[2] Klimsiak, T., Rzymowski, M.: Mokobodzkis intervals: An approach to Dynkin games when value process is not a semimartingale. *Stochastic Process. Appl.* **191** (2026) 104786