

12:25-1:00 Katharina Klioba (TU Delft, Netherland)

Milstein-type schemes for hyperbolic SPDEs

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Discretisations of SPDEs with multiplicative noise in time that rely solely on the Wiener increments $W(t_{j+1}) - W(t_j)$ are known to converge at most at rate $1/2$. To overcome this limitation, one needs to consider higher-order terms from an Itô-Taylor expansion, leading to iterated stochastic integrals. The resulting Milstein scheme is well-studied for both SDEs and parabolic SPDEs. In this talk, convergence rates of the Milstein scheme are presented for hyperbolic SPDEs, where no smoothing effect over time can be leveraged. Optimal convergence rates are derived for the pathwise uniform strong error

$$E_h^\infty := \left(\mathbb{E} \max_{1 \leq j \leq M} \|U(t_j) - u_j\|_X^p \right)^{1/p}$$

on a Hilbert space X for $p \in [2, \infty)$. Here, U is the mild solution and u_j its Milstein approximation at time $t_j = jh$ with step size $h > 0$ and final time $T = Mh > 0$. For sufficiently regular nonlinearity and noise, we establish strong convergence of order one, with the error satisfying $E_h^\infty \lesssim h\sqrt{\log(T/h)}$ for rational Milstein schemes and $E_h^\infty \lesssim h$ for exponential Milstein schemes. This extends previous results from parabolic to hyperbolic SPDEs, from exponential to rational Milstein schemes, and from root-mean-square error estimates to pathwise uniform estimates. Applications include the stochastic Schrödinger and transport equations. This is joint work with Felix Kastner.