10:10-10:45 Irene Tubikanec (Johannes Kepler University Linz, Austria)

Splitting methods for stochastic Hodgkin-Huxley type systems

Pierre Étoré (University Grenoble Alpes), Anna Melnykova (University of Avignon), Irene Tubikanec (Johannes Kepler University Linz)

In this talk, we consider a class of locally Lipschitz SDEs of Hodgkin-Huxley type, characterized by a conditionally linear drift structure. For these systems, we construct different Lie-Trotter and Strang splitting methods exploiting their conditional linearity. In contrast to Euler–Maruyama type methods, the proposed splitting schemes are inherently structure-preserving. In particular, we prove that they are geometrically ergodic and preserve the natural state space of the model. Numerical experiments demonstrate that the splitting schemes are markedly superior in preserving the qualitative dynamics of neuronal spiking and allow for significantly larger time steps. While structure preservation follows naturally from the construction of the splitting methods, establishing mean-square convergence is considerably more challenging. Existing fundamental theorems for mean-square convergence of numerical methods for SDEs require globally or one-sided Lipschitz continuous coefficients, while strong convergence results under merely local Lipschitz conditions are largely restricted to Euler–Maruyama type methods. To address these limitations, we introduce a new localized version of the fundamental mean-square convergence theorem for SDEs with locally Lipschitz coefficients. Specifically, we show that if a numerical scheme is locally consistent in the mean-square sense of order $q > 1$, then it is locally mean-square convergent with rate $q - 1$. Building on this result, we further prove that global mean-square convergence (without rate) follows, provided that both the exact solution and its numerical approximation admit bounded moments. This convergence framework is then applied to the splitting schemes, requiring innovative proofs of local consistency and boundedness of moments.